Pion Spectra in the Production of Resonances by Neutrinos

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Abstract

A method is presented using helicity cross sections for calculating neutrino-nucleon interactions. The formalism is applied in the calculation of the pion spectra produced by ν_μ and ν_τ beams. The masses of the charged leptons are kept throughout the calculations. Cross sections are presented in numerous figures where the contributions of the significant form factors are also shown. The article describes the steps of the calculation and gives details so that it can be reproduced and adapted to the kinematic conditions of the experiments.

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I. INTRODUCTION

Neutrino production of resonances is attracting a lot of attention because differential cross sections will be measured in a new generation of experiments which will try to verify the functional form of the cross sections *i.e.* the number and the Q^2 dependence of form factors(FF). They will also be used as an input to study properties of neutrinos in oscillation experiments. The dominant signal at low neutrino energies will be the Δ-resonance. Many of the completed experiments detected the energy and the angle of the produced muon which motivated theoretical authors to integrate over the phase space of the decay products, thus presenting cross sections $\frac{d\sigma}{dQ^2}$, $\frac{d\sigma}{dW}$ and the integrated cross section. The comparisons of the calculated cross sections (differential or integrated) are consistent with the data, but we must confess the error bars are large so that there is a significant spread on the experimental points.

The accuracy will improve in the new experiments and explicit distributions on the energy spectrum of the pions produced in the decays will become available. This motivates us to calculate the pion spectrum from the diagram in Fig. 1 by keeping the Δ-propagator and without integrating over the whole phase space of the pion. We shall present the calculation in detail so that the interested reader can reproduce and use our results. We will also make available a code for our calculation which can be used by experimentalists.

The method that we adopt decomposes the leptonic tensor into helicity components and uses helicity cross sections for the scattering of the $W^{\pm}$ or Z^0 bosons on the nucleons. This method has been found to be useful [8, 9] and was adopted recently in the coherent pion production by neutrinos [10]. This way the calculation, whose algebra is long and tedious, simplifies. We decided to present results for free protons and neutrons in order to show their main features and separate them from nuclear target effects. We also take the opportunity to mention and correct a mistake in the pion spectrum that appears in an earlier article on which one of us (EAP) was a co-author.

In addition to describing the formalism we use it for the calculation of the differential and integrated cross sections. We calculate the energy spectra of produced pions by ν_{μ} and ν_{τ} beams. We keep the masses of the charged leptons throughout the calculations or set them equal to zero in order to see the changes brought about in the spectra. We also show in many figures the contributions of the important form factors and their interferences explicitly. Section II describes the method and includes detailed formulas for the cross sections. The functional dependence of the form factors are included in section III, where they are used in order to calculate the results in figures 2-10. A summary of the results and of the improvements that have taken place over the past few years are included in the last section.

II. THE METHOD

The process we consider is in general

$$\nu(\vec{k})\, N(\vec{p}) \rightarrow \mu^-(\vec{k}')\mathcal{R}(\vec{p}_\Delta) \rightarrow \mu^-(\vec{k}')\, \pi(p_\pi)\, N(p') \tag{2.1}$$

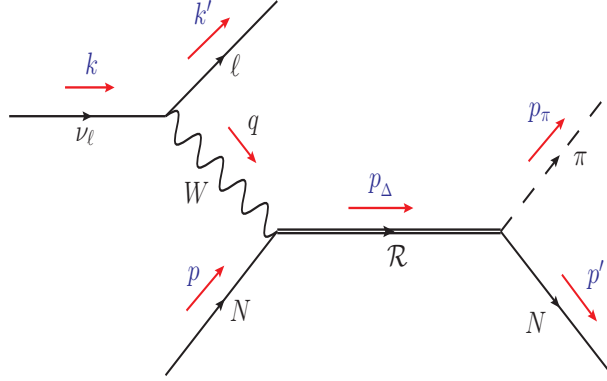with $\mathcal{R}$ being a resonance with spin $3/2$. It is convenient to use variables in the rest frame

FIG. 1: *Diagram of the process*

of the nucleon

$$q = k - k', \quad Q^2 = -q^2, \quad W^2 = p_\Delta^2, \quad \nu = E - E' \tag{2.2}$$

inspired from the kinematics of deep inelastic scattering. The leptonic tensor is

$$\mathcal{L}_{\mu\nu} = 4\left[k_\mu k'_\nu + k_\nu k'_\mu + g_{\mu\nu}k.k' - i\epsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta\right] = \sum_{h,h'} L_{hh'}\varepsilon_{h'}^{\mu*}\varepsilon_h^\nu \tag{2.3}$$

which can be decomposed in terms of the polarizations of the exchanged current. When we keep the mass of the muon or tau lepton, there are polarizations for the spin-1 and zero states. In the laboratory frame we introduce the basis vectors

$$\varepsilon_R^\mu = \frac{1}{\sqrt{2}}\,(0,1,i,0)$$

$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}\,(0,1,-i,0)$$

$$\varepsilon_0^\mu = \frac{1}{\sqrt{Q^2}}\,(|\vec{q}|,0,0,q_0) \tag{2.4}$$

for helicities and the scalar component

$$\varepsilon_\ell^\mu = \frac{q^\mu}{\sqrt{Q^2}}. \tag{2.5}$$

We warn the reader that various definitions occur in the published articles which differ from each other. For instance, the above notation is slightly different from than in ref. [10]. Numerous articles in the early studies of resonance production and recently [11, 12] define a set of polarizations in the rest frame of the resonance because it simplifies calculations. The above set is convenient with the first three polarizations being present even when the leptons are massless and the longitudinal component appearing for massive leptons. It is also a complete and orthonormal set of polarizations.

The coefficients $L_{h'h}$ are obtained by inverting Eq. 2.3

$$L_{h'h} = \mathcal{L}_{\mu\nu}\varepsilon_{h'}^{\mu*}\varepsilon_h^\nu. \tag{2.6}$$

When we average over the azimuthal angles of the produced hadrons, only the diagonal elements of the density matrix, as well as the $\ell 0$ interference term survive in the cross section. They were calculated in ref. [10] and we give them again for completion:

$$
\begin{aligned}
L_{RR} &= \frac{Q^2}{|\vec{q}|^2}(2E - \nu + |\vec{q}|)^2 - \frac{m_\mu^2}{|\vec{q}|^2}\left[2\nu(2E - \nu + |\vec{q}|) + m_\mu^2\right] \\
L_{LL} &= \frac{Q^2}{|\vec{q}|^2}(2E - \nu - |\vec{q}|)^2 - \frac{m_\mu^2}{|\vec{q}|^2}\left[2\nu(2E - \nu - |\vec{q}|) + m_\mu^2\right] \\
L_{00} &= \frac{2\left[Q^2(2E - \nu) - \nu m_\mu^2\right]^2}{Q^2|\vec{q}|^2} - 2(Q^2 + m_\mu^2) \\
L_{\ell\ell} &= 2m_\mu^2\left(\frac{m_\mu^2}{Q^2} + 1\right) \\
L_{\ell 0} &= \frac{2m_\mu^2[Q^2(2E - \nu) - \nu m_\mu^2]}{Q^2|\vec{q}|}
\end{aligned}
\tag{2.7}
$$

All matrix elements are positive in the physical region, which becomes evident when the kinematic condition $Q_{min}^2 = m_\mu^2 \frac{\nu}{E-\nu}$ is taken into account. For the propagator of the spin-3/2 resonance we introduce the Rarita-Schwinger propagator in free space [13]

$$
\begin{aligned}
G_{\mu\nu}(p_\Delta) &= \frac{\not{p}_\Delta + M_\Delta}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta}\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3}\frac{1}{M_\Delta^2}p_{\Delta\mu}p_{\Delta\nu} + \frac{1}{3}\frac{1}{M_\Delta}(p_{\Delta\mu}\gamma_\nu - p_{\Delta\nu}\gamma_\mu)\right] \\
&\equiv \frac{\not{p}_\Delta + M_\Delta}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta}G'_{\mu\nu}
\end{aligned}
\tag{2.8}
$$

which is sufficient for the present work. The mass of $M_\Delta = 1232\text{MeV}$ and the width $\Gamma_\Delta = 120\text{MeV}$ will be taken as constants. But in some articles it is a function of the invariant mass $\Gamma_\Delta = \Gamma_{\Delta 0}\left(\frac{p_\pi(W)}{p_\pi(M_\Delta)}\right)^3$ with $p_\pi(W) = \frac{1}{2M_\Delta}\sqrt{(W^2 - M_N^2 - m_\pi^2)^2 - 4M_N^2 m_\pi^2}$. Several authors studied the modifications of the propagator in nuclear matter, which will be useful when we consider nuclear corrections.

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The matrix element for the entire process includes the coupling of the $Wp\Delta$-vertex given in terms of form factors and the $\pi p\Delta$ coupling $ig_\Delta p_\pi^\mu$. The matrix element is

$$\mathcal{M}_h = \sqrt{3}\ ig_\Delta p_\pi^\mu\ \bar{u}(p')\left[G_{\mu\nu}(p+q)\ d^{\nu\lambda} + \frac{1}{3}G_{\nu\mu}(p-p_\pi)\ d^{\lambda\nu}\right]u(p)\ \varepsilon_\lambda(q,h) \qquad (2.9)$$

For our process there are two propagators; one in the s-channel for Δ^{++}, as shown in figure 1, and one in the u-channel for Δ^0. The arguments of the propagator are $(p+q)^2$ and $(p-p_\pi)^2$, respectively. The argument of the polarizaion is q_μ and h denotes helicity. In this preprint we include, for the calculation and the curves, only the s-channel pole which resonates. When we submit the article for publication both terms will be included.

The coupling at the $Wp\Delta$-vertex are included in $d^{\lambda\nu}$ which will be discussed below. The $\pi p\Delta$ coupling is taken from Appendix A1 of ref. [14]: $g_\Delta = 15.3\text{GeV}^{-1}$. The factor $\sqrt{3}$ originates from the isospin relation

$$\langle\Delta^{++}|V_\mu^+|p\rangle = \sqrt{3}\langle\Delta^+|V_\mu^3|p\rangle \qquad (2.10)$$

where the right hand side is related to the electromagnetic form factor, whose numerical value was determined in early experiments and are used in many articles. Electroproduction data have been used as an input for neutrino reactions[1] and this convention still survives. The factor $1/3$ in front of the u-channel pole comes from the Clebsch-Gordan coefficients.

The $Wp\Delta$-vertex contains vector and axial form factor included in the function

$$d^{\nu\lambda} = g^{\nu\lambda}\left[\frac{C_3^V}{M_N}\slashed{q} + \frac{C_4^V}{M_N^2}p_\Delta.q + \frac{C_5^V}{M_N^2}p.q + C_6^V\right]\gamma_5 - q^\nu\left[\frac{C_3^V}{M_N}\gamma^\lambda + \frac{C_4^V}{M_N^2}p_\Delta^\lambda + \frac{C_5^V}{M_N^2}p^\lambda\right]\gamma_5$$
$$+ g^{\nu\lambda}C_5^A + q^\nu q^\lambda\frac{C_6^A}{M_N^2} \qquad (2.11)$$

The vector form factors were determined [14] using electroproduction data. Among the axial form factor the most important are $C_5^A(q^2)$ and $C_6^A(q^2)$ and for this reason we omitted the other two axial form factors. All form factors will be given explicitly in the next section. The square of the matrix element for the s-channel pole is

$$\mathcal{M}^{h'*}\mathcal{M}^h = \frac{3}{2}\frac{g_\Delta^2}{(p_\Delta^2 - M_\Delta^2) + M_\Delta^2\Gamma_\Delta^2}Tr[\ p_\pi^\mu(\slashed{p}_\Delta + M_\Delta)G'_{\mu\nu}d^{\nu\lambda}\varepsilon_\lambda(h)(\slashed{p} + M_N)$$
$$\varepsilon_{\lambda'}^*(h')d^{\nu'\lambda'}G'_{\mu'\nu'}(\slashed{p}_\Delta + M_\Delta)p_\pi^{\mu'}(\slashed{p}' + M_N)](2.12)$$

The factor $1/2$ comes from averaging over initial spins of the target. The helicity cross sections and interference terms for the scattering of the current on a proton target are

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[1] To our knowledge the factor $\sqrt{3}$ was introduced first by Schreiner and von Hippel and has become traditional to keep it in recent calculations.

defined as

$$\frac{d\sigma^{h'h}}{dE_\pi}(\nu, Q^2) = \frac{1}{32\pi\nu M_N |\vec{p}_\Delta|} \mathcal{M}^{h'*} \mathcal{M}^h \tag{2.13}$$

We now have all the ingredients for calculating helicity cross sections for the processes $W^+ p \to \mathcal{R}^{++} \to \pi^+ p$. The calculation is straight forward since it involves a two-body phase space and a trace. It is long because of the many γ-matrices occurring in the propagator and the $Wp\Delta$-vertex. The trace calculation was done using FEYNCALC [15].

Finally we can include the lepton variables and present the triple differential cross section

$$\frac{d\sigma}{dE_\pi \, dQ^2 \, d\nu} = \frac{G^2}{4\pi^2} \frac{\nu}{4E^2} |V_{ud}|^2 \left[L_{00} \frac{d\sigma^S}{dE_\pi} + L_{LL} \frac{d\sigma^L}{dE_\pi} + L_{RR} \frac{d\sigma^R}{dE_\pi} + L_{\ell\ell} \frac{d\sigma^\ell}{dE_\pi} + 2 L_{\ell 0} \frac{d\sigma^{\ell 0}}{dE_\pi} \right] \tag{2.14}$$

This formula includes the muon mass contained in the $L_{h'h}$ functions. In the limit $m_\mu = 0$ it reduces to the known result [8]. We note that the formalism simplifies the calculations because the leptonic part was incorporated as an overall factor. We also note that there is only one interference term $\frac{d\sigma^{\ell 0}}{dE_\pi}$ because the other interference terms vanish when we average over the azimuthal angle of the produced hadrons.

III. NUMERICAL ESTIMATES

Besides the form factors we have now all the ingredients for calculating the pion spectrum. The vector form factors have been studied in earlier paper determining their Q^2-dependence from electroproduction data. It has been established that they are modified dipoles

$$C_3^V(Q^2) = \frac{C_3^V(0)}{(1 + Q^2/M_V^2)^2} \frac{1}{1 + Q^2/(4\,M_V^2)} \tag{3.1}$$

with $C_3^V(Q^2 = 0) = 1.95$ and $M_V = 0.84$GeV. The dominance of the magnetic dipole gives the relation

$$C_4^V(Q^2) = -C_3^V(Q^2) \frac{M_N}{W}, \quad C_5^V = 0. \tag{3.2}$$

The other two terms $C_5^V = C_6^V$ were set to zero. These form factors were determined by electroproduction data where it was shown that they reproduce the measured helicity amplitudes [14].

The axial couplings were obtained from the decay rate of the Δ-resonance and the reproduction of neutrino data

$$C_5^A(Q^2) = \frac{C_5^A(0)}{(1 + Q^2/M_A^2)^2} \frac{1}{1 + Q^2/(3\,M_A^2)} \tag{3.3}$$

with $C_5^A(Q^2 = 0) = 1.2$ and $M_A = 1.05$GeV. PCAC gave us

$$C_6^A(Q^2) = C_5^A(Q^2) \frac{M_N^2}{Q^2 + m_\pi^2}. \tag{3.4}$$

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$C_6^A(Q^2)$ is the pseudoscalar form factor with its contribution to the cross section being proportional to the square of the lepton mass.

The triple differential cross section $\frac{d\sigma}{dE_\pi\, dQ^2\, dW}$ shown in figure 2 for neutrino energy of 1.0GeV, $E_\pi = 300$MeV, $Q^2 = 0.2, 0.5$ and 0.8GeV^2. The curves show a Δ-peak which is a sensitive function of Q^2. This is expected since for large values of Q^2 there is the large decrease of the form factors. Integrating over W one obtains the double differential cross section of figure 3. Again the cross section decrease with increasing E_π because the process runs out of phase space.

Finally more interesting is the dependence on the pion energy when all other variables are integrated. Figure 4 shows the contribution of the important form factors. the term C_5^A dominates with the next contribution coming from C_3^V. The interference between C_3^V and C_4^V is destructive. The interference between vector and axial form factors is constructive for neutrinos and destructive for anti-neutrinos. In figure 5 we set the muon mass equal to zero in order to see the effect of neglecting the mass. We repeat the calculations for higher neutrino energies $E_\nu = 1, 2$, and 5 GeV shown in figures 6-8.

The mass of the charged lepton influences the pion spectra shown in Figures 4 and 5. Mass effects are much more for ν_τ beams, where the threshold effect is dominant. In Fig. 9 we show the pion spectrum for $E_{\nu_\tau} = 5$GeV.

A new feature is the change in the significance of the various form factors. The induced pseudoscalar form factor C_6^A is more important relative to C_5^A. The integrated cross-section is smaller because it reaches its asymptotic value at a much higher energy. In Fig. 10 we show the integrated cross-section as a function of E_{ν_τ}. In all the figures, we have included the spectra for anti-neutrinos, which are obtained by changing the sign of the vector $\otimes$ axial interference terms. In closing, we remark that the pion spectra show relevant new features, and will be important in deciphering the importance and the functional dependence of the form factors.

For comparison with other articles one must keep in mind that we did not include nuclear effects from the target. We decided to use protons or neutrons as free targets in order to study the significance of the various form factors. We may include nuclear target effects later on.

Several articles calculated and presented the pion spectrum and we comment on them. An early article [2], where one of us is a co-author, presented in figures 8-16 pion energy spectra with a different shape because the phase space was treated incorrectly. The discrepancy was noticed and corrected in figures 3 and 4 of ref. [6]. The same discrepancy

has been pointed out when the spectrum was calculated in [7]. Between the previous two articles, mentioned above, and the present article there is a difference in the method of calculation. The earlier articles calculated the production of the delta resonance and then folded its decay into a pion and a nucleon. This method however requires knowledge of the density matrix elements as described in Equs (1.3) and (1.4) of ref. [16], because resonances in various polarization states produce pions with different energies. In the present article we calculate the entire process with the delta resonance in the intermediate state. The same procedure is advocated in a recent article [5]. Thus the pion energy spectrum on a Hydrogen target is a rather interesting quantity being sensitive to the form factors.

IV. SUMMARY

Neutrino interactions are reaching an age of theoretical maturity and will come to be compared with the new generation of experiments. For this reason we investigated the neutrino cross-sections in the energy range of the delta-resonance. To this end we have rewritten the neutrino-nucleon cross-section in terms of helicity of cross-sections of the $W^{\pm}$ on nucleons. For the sake of brevity, we have not included neutral current reactions, which will be presented in the future. Our plan is to present these results in a code and explicit publications.

Besides the formalism, we have made the following improvements:

1. We included the charged lepton mass. The reader who wishes to see effects originating form the mass can set them equal to zero in Eqs. 2.7 and include them in the phase space of the two-body cross-section given in Eq. 2.13. Such a comparison was presented in Figs. 4 and 5.

2. We use a running width for the resonance as described after Eq. 2.13. This was also included in ref. [14].

3. We study the significance of the pseudo-scalar form factor C_6^A for muon and tau neutrino-induced reactions. We have also presented contributions from the various form factors.

With this article, we hope to clarify several questions presented by colleagues who are planning and carrying out the experiments. There are several other quantities that need to be calculated, and for just this reason, we have developed a flexible formulation which can be adapted to new situations which may arise. Finally, as previously mentioned, we are preparing a CODE which will cover new demands that may come up in the future.

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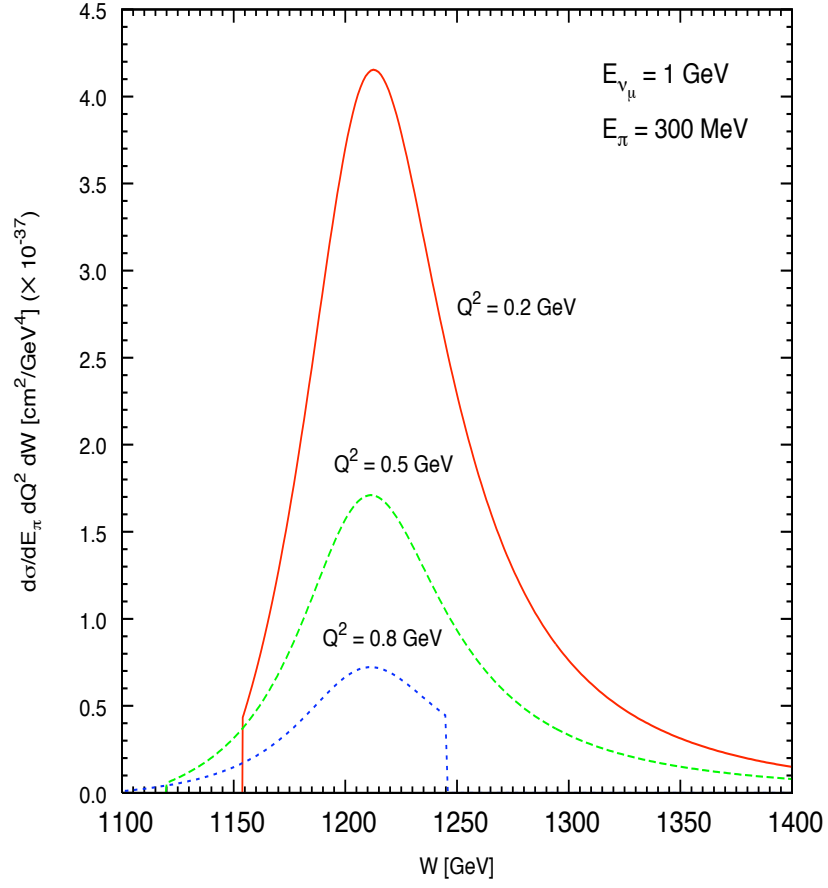[16] P. A. Schreiner and F. Von Hippel, Nucl. Phys. B **58**, 333 (1973).

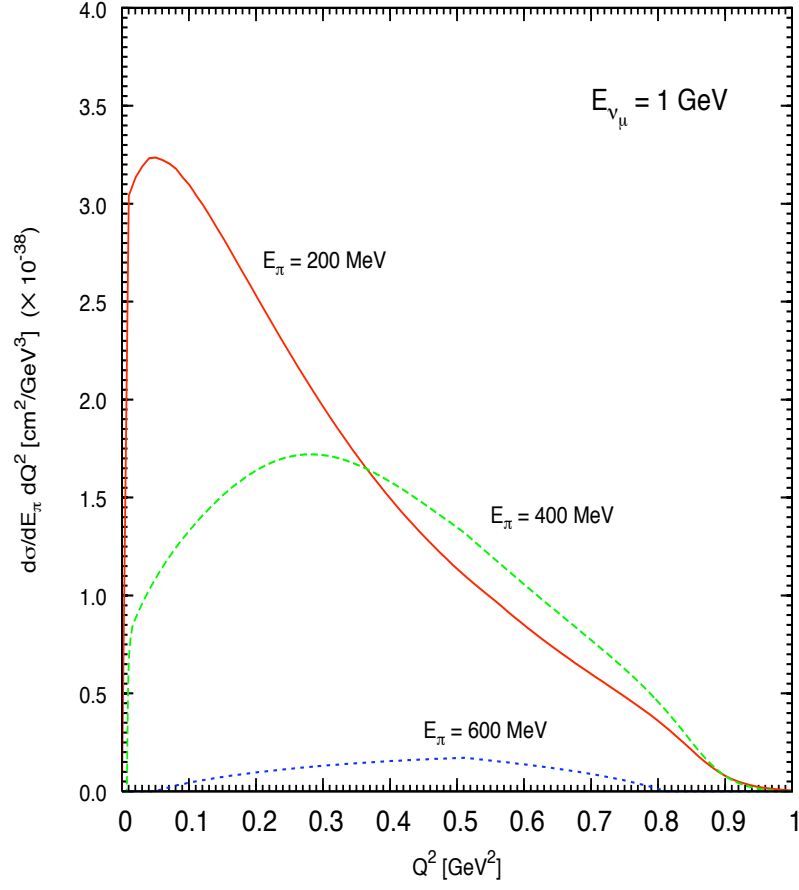FIG. 2: *Triple differential cross section for a 1 GeV ν_μ interacting with a proton for different Q^2.*

FIG. 3: *Double differential cross section for a 1 GeV ν_μ interacting with a proton for different E_π. This is calculated from Fig.?? after W integration.*
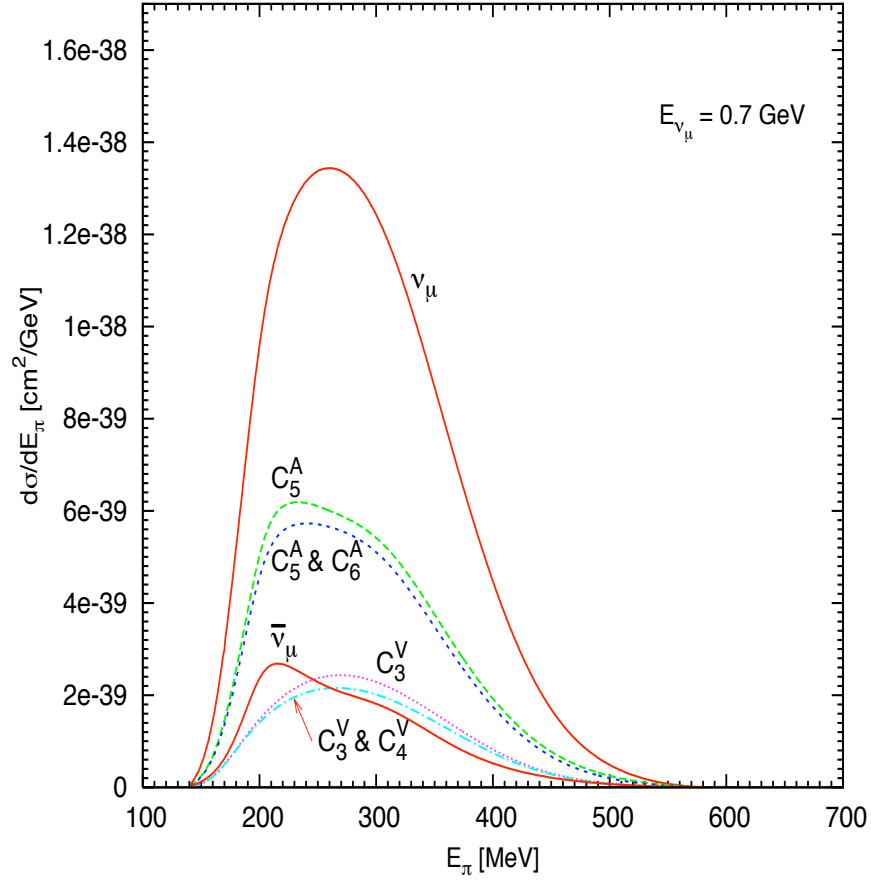
FIG. 4: *Pion energy spectrum for an incoming ν_μ of energy* 0.7 *GeV. Contributions from several from factors are shown separately. Here, for example, $C_5^A \& C_6^A$ means in this case we have put only these two form factors finite. Hence it contains their interference term as well. By ν_μ and $\bar{\nu}_\mu$ we denote the contributions of all the form factors to cross sections for these particles. Here muon mass effects are taken into account. For ν_μ a constructive interference is observed, while for $\bar{\nu}_\mu$ it is destructive.*
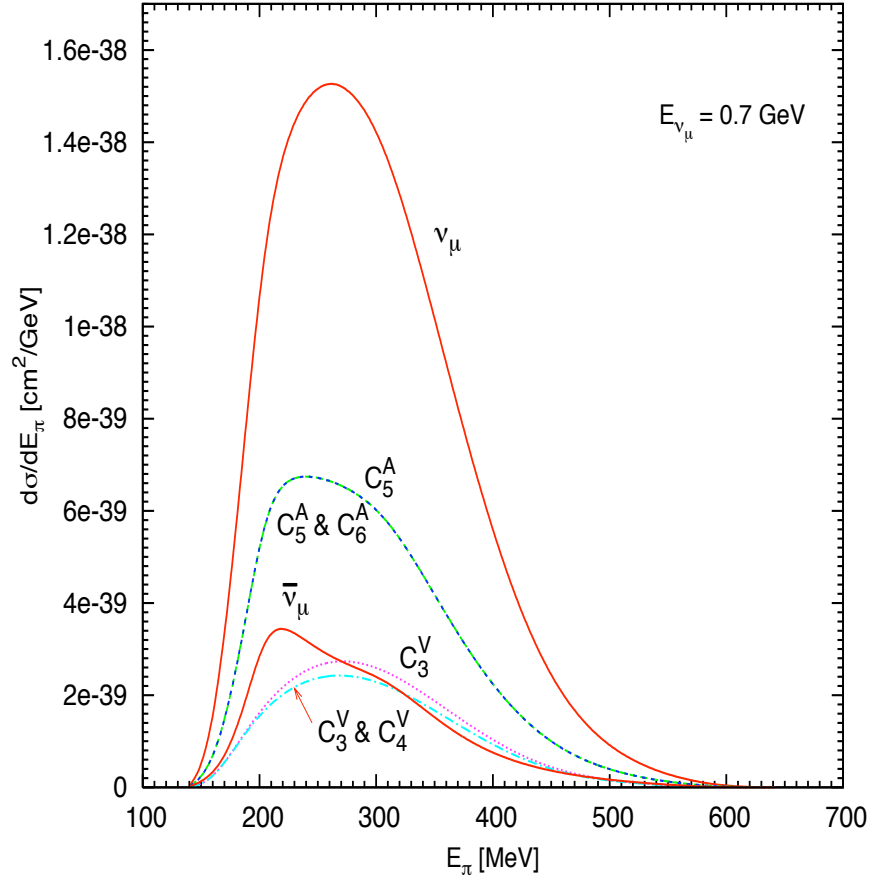
FIG. 5: *Same as Fig. 4 but with outgoing muon mass neglected. In this limit C_6^A does not contribute. These cross sections are a bit enhanced compared to Fig. 4 due to more phase space in this limit.*
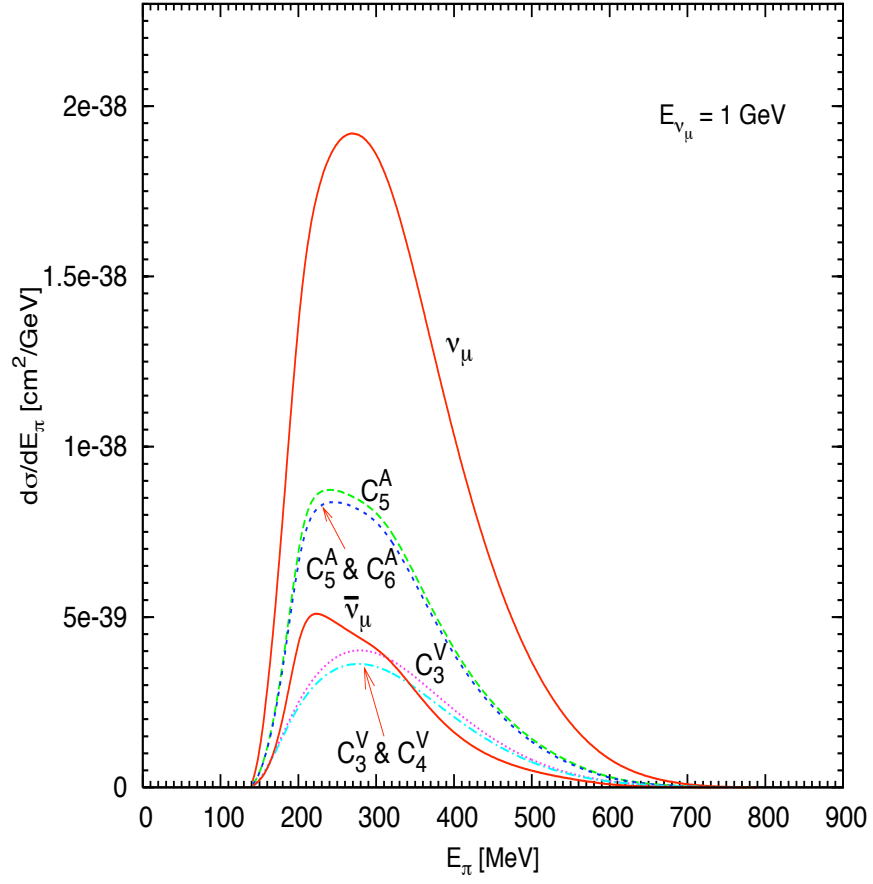
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FIG. 6: *Pion energy spectrum for an incoming ν_μ of energy 1 GeV. Notations are similar to Fig. 4.*
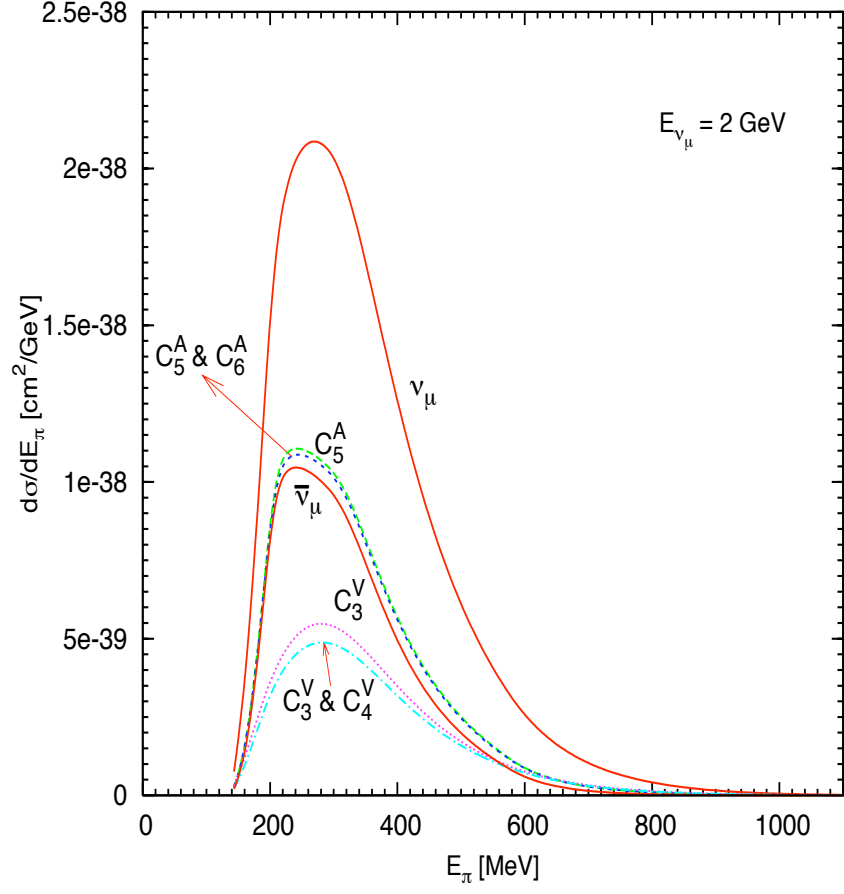
FIG. 7: *Pion energy spectrum for an incoming ν_μ of energy 2 GeV. Notations are similar to Fig. 4.*
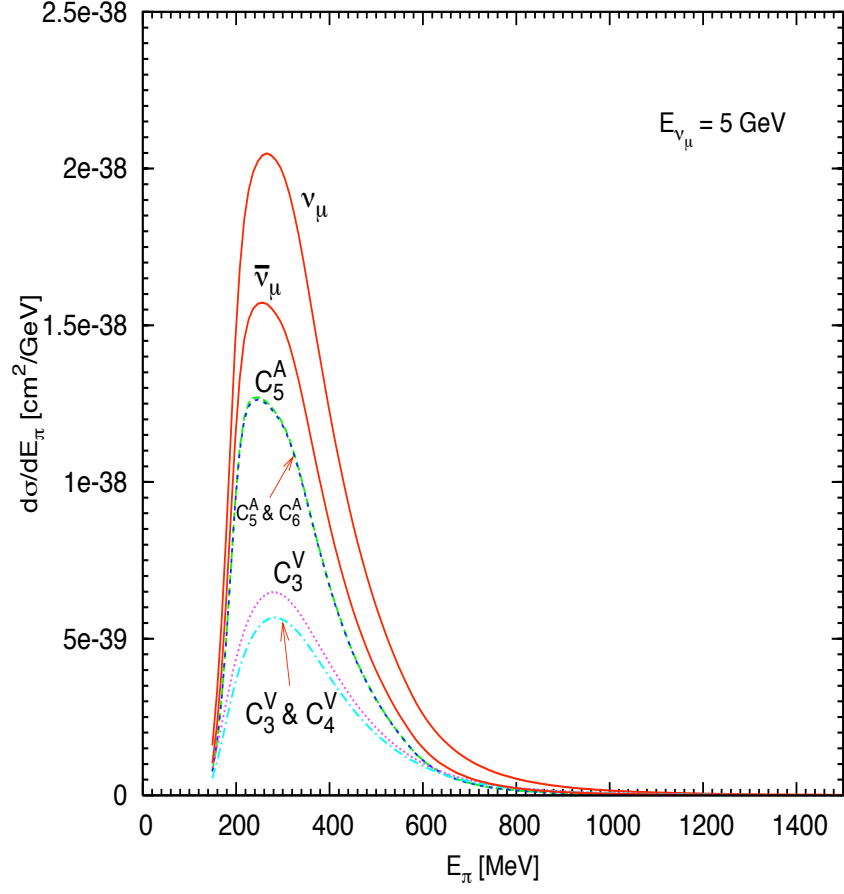
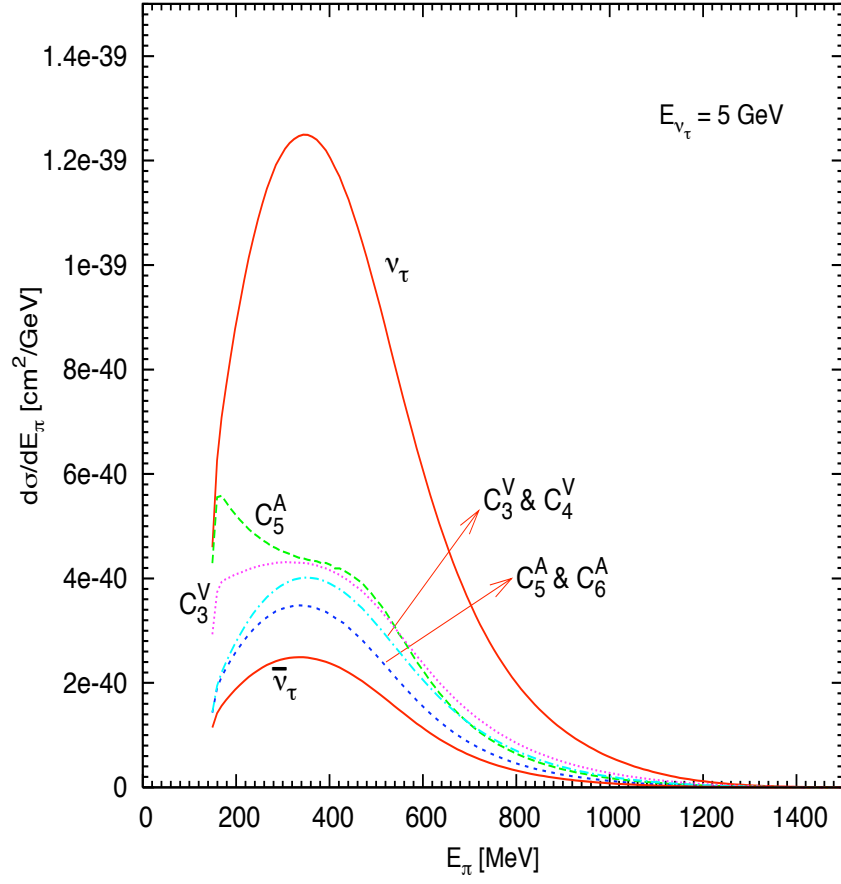FIG. 8: *Pion energy spectrum for an incoming ν_μ of energy 5 GeV. Notations are similar to Fig. 4.*

FIG. 9: *Pion energy spectrum for an incoming ν_τ of energy 5 GeV. Notations are similar to Fig. 4.*
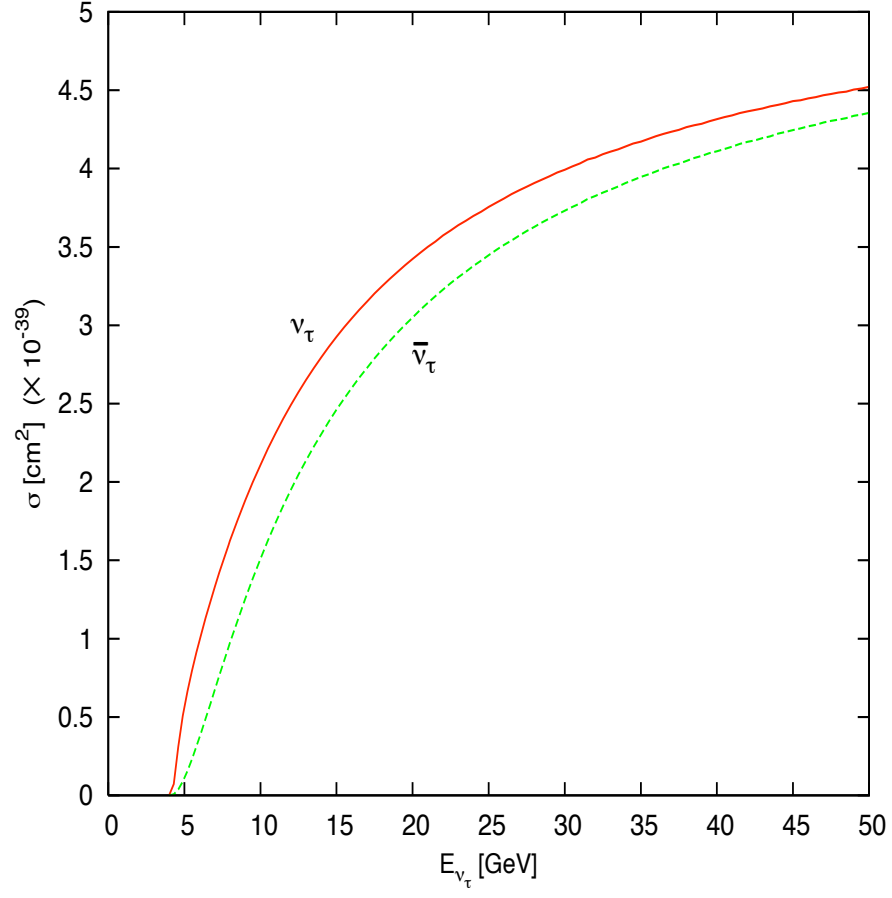
FIG. 10: *Variation of ν_τ total cross-section with neutrino energy.*